Write your name and student number in the top left corner of each page

1. Determine the z-transform of the following sequences and their respective ROCs including the zeros and poles (if applicable):

   (a) $x[n]=\left(\frac{1}{3}\right)^n \mu[n]$

   (b) $x[n]=\left(\frac{1}{3}\right)^n (\mu[n]-\mu[n-10])$

   Determine the inverse z-transform for the following functions:

   (c) $X(z)=\frac{1}{1+0.5z^{-1}}, \quad |z|<0.5$ anticausal

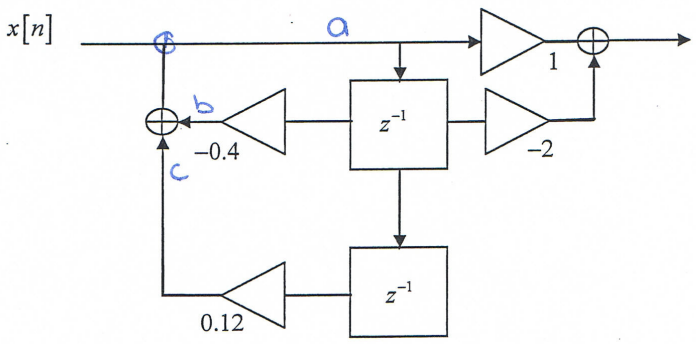
   (d) $X(z)=\frac{1}{1+0.5z^{-1}}, \quad |z|>0.5$

2. A continuous-time sinusoidal system $x_a(t)=A\cos(\Omega_0 t+\varphi)$ can be uniquely recovered from its sampled version $x[n]=x_a(nT)=\cos(\Omega_0 nT)$. For what values of T is $x[n]$ a periodic sequence? What is the fundamental period of $x[n]$ if $\Omega_0 = 20$ radians and $T=\pi/8$ seconds?

   $\cos\left(\frac{20\pi}{8}n\right)$

   $\frac{1}{40}$

3. Consider the following LTI system:



   Ω_0 = radians.

   (a) write down the transfer function of the system $H(z)=\frac{Y(z)}{X(z)}$

   (b) determine the ROC, poles and zeros and plot their positions in a zero-pole plot

   (c) what kind of a filter is this?

$3z-\left(\frac{1}{32}\right)^4=0$

$\frac{1-2z^{-1}}{1+0,4z^{-1}-0,12z^{-2}}$

4. Suppose we have two four-point sequences $x[n]$ and $h[n]$ as follows:

$$x[n] = \cos(\pi n / 2), \quad n = 0,1,2,3 \quad \text{and} \quad h[n] = 2^n, \quad n = 0,1,2,3$$

(a) calculate the four point DFT $X(k)$
(b) calculate the four point DFT $H(k)$
(c) calculate the circular four point convolution $y[n] = x[n] \otimes h[n]$ directly
(d) calculate $y[n]$ by multiplying $X(k)$ and $H(k)$ and performing the inverse DFT

5. The z-transform of a causal sequence $h[n]$ is given by:

$$H(z) = \frac{z(z+2.0)}{(z-0.2)(z+0.6)}$$

Determine the causal sequence $h[n]$ by partial-fraction expansion of $H(z)$

6. ***Bonus Question***

(a) Discuss the advantages and disadvantages of IIR digital filter systems as compared To FIR digital filter systems

(b) Describe how to use the wide knowledge of analog IIR filters for digital filter design